

# MATH HIGH SCHOOL: A TEACHING PROPOSAL

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## ABSTRACT

The purpose of this paper is to describe Math High School (MHS), an experimental teaching project promoted by the research group of Mathematics Education at the University of Salerno. MHS was devised to surpass the dichotomy between scientific and humanistic cultures, responding to the daily, social, politic, national and world challenges. In this process, Mathematics is the common denominator between the two cultures. In this paper we describe the theoretical context of MHS, the educational innovations of the methodologies and the first year's experimentation activities.

## 1 Introduction

Math High School (MHS) project has its doctrinal roots on postmodern philosophical ideas in mathematics education; it embraces the educational program associated to the theory of complexity of E. Morin. It consists of additional in depth lectures with respect to regular school mathematics classes, aimed at expanding the educational level of the students in order to improve the development of critical skills and aptitude to scientific research (Morin, 1993).

MHS is based on nietzschian philosophical theories, with the belief of a school education tending to enlarge as much as possible the student knowledge, in order to develop critical abilities and the aptitude for scientific research. Nietzsche believed that a good humanistic culture does not prevent the scientific knowledge and vice versa. He hypothesized the creation of a free school where more creative, but therefore weaker, spirits, can find a space to develop their potentialities and art, Nietzsche (1872). Thus, the school in general and Mathematics in particular must offer the opportunity to human being of understanding both universal external world and its most interior part. In particular, he hypothesizes the birth of a organic structure, the "Gaia Scienza", where the art melts with the science through Mathematics, which becomes the connection between the scientific "will to truth" and the artistic desire of illusion, Tortoriello (2015).

One of the main ideas of MHS is the belief in investing in education by competences, aiming to promote a coordinate system of knowledge and abilities moved by the subject for a purpose (a task, a set of tasks or an action) supporting a good internal motivational and sentimental aptitude (Pellerey, 2003). As mentioned by D'Amore, competences cannot be reduced to a single subject; they presume and create knowledge and suggest new uses and masteries, i.e. "competences generate competences" (D'Amore, 2000). The objectives are to

develop in students essential basic competences in a cultural education of citizens responding to ethic and social needs: suppose and solve problems, design and build models of real situations, adequately express information, grasp and imagine, make connections between different kinds of knowledge. The idea is of supplying contents usefully applicable also and especially outside the school's world, in daily life, as citizens rather than students: "competences must be an experience for the citizen (as abilities to solve problematic situations rather than knowledge, being able to choose proper resources, strategies and ways of thinking) (Arzanello, Robutti, 2002). Thus, it is needed to detect important contents constituting the basic core and then surround them by other supplementary contents. In Mathematics, besides the transmission of contents, it is needed to manage an aware and active reworked version of them, driven by motivation and volition, in order to allow their use and interpretation in problematic situations and to master the connections between different contents. When the student goes out of the usual classroom life, making connection between different pieces of knowledge, the idea of passing from mere knowledge to competence arises (Sbaragli, 2011).

Within the school of the competences, the educational purposes assume a social relevance: students must get the aptitude to organizing the knowledge. Not only is the teacher responsible of a correct acquisition from the students, but also of "teaching" involving emotional and motivational aspects. Moreover, he has the task of "educate": in this way, the school becomes education of human condition, apprenticeship to life, apprenticeship to uncertainty, education to European and global citizenry (Morin, 2000). Education by competences aims to surpass the fragmented knowledge of multidisciplinary on the side of pluridisciplinarity, which means that the same topic is proposed from the point of view of various disciplines through a planning by disciplinary environments. The transfert of knowledge in a multi-objective education by competences is realized through an education which surpasses also the pluridisciplinarity with interdisciplinarity. Nevertheless, the actual education revolution is realizable only with a transdisciplinary setting. As mentioned by Mauro Laeng, the term "transdisciplinary" describes the "interdisciplinarity in the strong sense", because at this level "the actual surpass of an epistemological barrier with the discovery of a new unite horizon" happens (Laeng, 1992).

An education by competences expects a bind between humanistic and scientific cultures, finding the most natural contextualization in historical research of mathematical contents and in their revival under a modern interpretation, following the last guidelines of the Italian high school reform. On the definition of a common European framework of qualifications and scholastic competences (G.U.E., 2008), the reform indicates in the connection between "scientific and humanistic cultures, ..., through laboratorial practice" one of the main features of new lyceums (Miur, 2010): "students should have numerous and varied experiences related to the cultural, historical and scientific evolution of mathematics so that they can appreciate the role of mathematics in the development of our contemporary society and explore relationships among mathematics and the disciplines it serves: the physical and life sciences, the social sciences, and the humanities. It is the intent of this goal-learning to value mathematics - to focus attention on the need for student awareness of the interaction between

mathematics and the historical situations from which it is developed and the impact that interaction has on our culture and our lives” (Swetz, 1995). Like Swetz suggests, we have done experiments in the measurement of  $\pi$ , using Eratosthenes technique to obtain the circumference of the earth, Greek ruler and compass construction, and algebraic equations. The benefit of any historical math is that there is a great opportunity to reinforce skills and provide opportunity for enrichment.

MHS was born from cultural and social motivations. If the cultural roots of this educational path are fed by theory of complexity, the social motivations need have to overtake the cultural gap between Italy and other European states, as in the statement of the OECD-PISA surveys results. Indeed, the competences of Italian teenagers in Mathematics are under the average of OCSE. In particular, the results in Mathematics are worse than the total average of the south of Italy, in general characterized also for a more internal variability of results: over the national average north west and north east are collocated, and the centre of Italy is on the mean. Thus, the results of south of Italy and islands are lower than the national average. The results from national INVALSI survey are quite similar to OECD ones. Indeed, “both for Mathematics and Literature, the rank of the single regions emerging from two sources is similar. Also coherent is the position of single school which have participated to both surveys” (OCSE-PISA Rapporto Nazionale, 2012). Thus, there is the immediate need of a revision in the entire national territory, but in particular in the south of Italy where the results have been worse. The general aim is to supply to students the tools to deal with the complexity of the current society. On the one hand, it is necessary to surpass the fragmented knowledge in the same way as it was requested from the Gentile’s reform of 1923. On the other hand, it is needed to increase the Mathematical Literacy, that “is an individual’s capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens” (Draft Mathematical Frameworks, 2015).

## **2 Math High School (MHS)**

MHS is a research and educational project, devised by the research group of Mathematics Education at University of Salerno. The schools adhering to the project have made an agreement with the Department of Mathematic, to find the experts covering the supplementary courses in mathematics, as expected by the proposed theoretical framework. It is composed of additional advanced courses of Mathematics and other disciplines to it related. The activities are programmed in the five years of high school. In particular, 40 hours are expected for the students of the first year, 60 hours for those of the second, 70 hours the third, 80 hours for the fourth and 80 hours for the fifth year. The courses occur at single institutes and in a span of about seven months, from November to May, with one lesson per week.

We structured each educative module in order to improve some Mathematics competences.

The time schedule is the following (Table 1).

Table 1. Time schedule and subject

Subject	1 <sup>st</sup> year	2 <sup>nd</sup> year	3 <sup>rd</sup> year	4 <sup>th</sup> year	5 <sup>th</sup> year
Mathematics and Italian Literature	0	5	15	15	15
Mathematics	10	10	10	15	15
Physics	10	10	5	10	10
Mathematics and Philosophy	0	0	10	10	10
Logic	20	25	10	10	10
Mathematics and History	0	5	10	10	10
Mathematics and Chemistry	0	0	5	5	5
Mathematics and Biology	0	5	5	5	5
<i>TOTAL</i>	<i>40</i>	<i>60</i>	<i>70</i>	<i>80</i>	<i>80</i>

### 3 Methodology

The MHS educative activities are inspired by constructivist learning. It is based on the active participation of the students to problem solving and it aims to the development of a critic thinking. The students “build” their own knowledge starting from a test and exploit their knowledge and previous experiences applying these new situations and adding intellectual constructs. In this constructivist education context, it has been also proposed, in some learning object, a flipped teaching (Hamdan et al., 2013) as a model of experimentation of the future classroom, using a revolution of the traditional methodologies, flipping the old system: an explanation time in classroom from the teacher, an individual study at home and a phase of test in classroom again. In this way, the time spent at school is more functional and productive to the teaching-learning process, investing the teaching hours to solve most complex problems, deepen topics, connect themes and analyzing the disciplinary contexts, produce team work in peer to peer mode in a laboratory driven context. The teacher becomes a driver and tutor, supplying his assistance in classroom to students in order to stress significant observations and considerations, through the use of exercises, shared researches and learning by doing.

Within the activated learning modules, the educative methodologies have been sharpened to cope with the objectives. We focused on the language and the skills improvement of students in debating, in order to ease the passage from an informal register to an advanced one (Ferrari, 2004). As observed by Bruner, for Vygotskij and Dewey every type of language is a way to self-manage our thoughts about reality and the thinking is a way to organize the perception and the actions (Bruner, 2005). In each proposed activity, the history of Mathematics played an important role and it has been applied in several fields: read aloud mathematical stories to the class, students write about mathematical history topics, students perform plays and skits or make videos about historical topics, hands-on experiences, through

the arts, through the visual arts (Reimer and Reimer, 1995). Students develop and refine estimation strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate. The involved artefacts have been meant as both technical instruments and psychological instruments, that are “semiotics mediation instruments” (Bartolini et al., 2006). We did not omitted the “centrality of the role of operator, of the person building its own concepts on the base of knowledge not deriving from an eventful discovery, but from thoughts, buildings, aware and voluntary activities of other people” (Boero & Garuti, 1994). Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonable of answers. Students reinforce skills with whole numbers, fractions and decimals through problem solving and application activities.

## **4 First Experimentations**

MHS project has been experimented in academic year 2014/2015, with 14-15 years old students, attending the first year of Liceo Scientifico “P.S. Mancini” in Avellino, Liceo Scientifico “P. P. Parzanese” in Ariano Irpino (AV) and Istituto Superiore “A. Gatto” in Agropoli (SA). The activated modules since now have been three: Mathematics, Logic, Physics.

### **4.1 Mathematics Module**

The Mathematics module has been meant as improvement of basic Mathematics knowledge, with particular reference to geometry and fundamentals of Mathematics:

- Euclidean geometry insights: Napoleon's Theorem, Torricelli configuration, Euler line, continuous triangles, pedal triangles and pedal quadrilaterals, orthic triangles and orthic quadrilaterals, Van Aubel's Theorem, analysis of the characteristic processes of human thought (axiomatizations, definitions, proofs, generalizations), equality between geometric figures.
- numeric sets  $N$ ,  $Z$ ,  $Q$ ,  $R$ : geometric model;
- basics in arithmetic: the principle of mathematical induction, recursive sequences, modular arithmetic, tests of divisibility, the finished structures; binomial coefficients; secrets of Pascal's triangle.

Some contents of geometry, such as those related to the construction with ruler and compass and other mathematical machines, were analyzed through a historical investigation and re-contextualized, also with the use of dynamic geometry software.

### **4.2 Logic Module**

The Logic learning module was organized focusing on the historical path of logic-mathematics birth, from the origins to the present. With this aim, every content of the course has been historically contextualized. The relation between the language and the development of logic abilities. We noticed that the level of linguistic competence influenced the problem solving since first interventions. In a vygotskian picture, we managed to analyze how the language supported the development of a logic thinking (Vygotskij, 1934). The study of competences of problem solving opens a window on the people capacity in cognitive

activities as basis for other cognitive approaches of general order, to face the life challenges (Lesh and Zaqojewski, 2007). In agreement with PISA 2012, we considered the “competence in problem solving” as the ability of an individual of starting cognitive processes to understand and solve problematic situations for which the solution is not clear from the beginning. This competence is comprehensive of the own will to face with those situations to realize own potentialities as thinking citizens and with a constructive role (OCDE, 2013). In order to develop the ability to solve problems, logic surveys have been proposed to students, organized in cooperative teams. To each group, we assigned the task of answering to the survey, agree on and motivate a final group answer. Being the problem solving competence dependent from knowledge and specific strategies by topic (Mayer, 1992; Funke and Frensch, 2007), we proposed surveys to students whose solution did not required previous knowledge.

### **4.3 Physics Module**

In the Physics module, the teaching was realized in laboratorial form, favoring the heuristic approach through an experimental-inductive method. Inquiry Based Science Education (IBSE) was the model, i.e. a teaching based on the investigation of problems, critical group discussion and search for new solutions in a constructivist perspective. We have proposed to students a problem from the observation of reality and asked them to identify it by formulating hypotheses (engage).

After that, they had to plan the survey exploring the variables (explore), they led the survey individually or in groups by documenting the results (explain). Then, together with the teacher who served as scaffolder in all the activities, the results (evaluate) and communicated by formulating new problems have been interpreted (extend). This investigative path of reality has encouraged the development of problem posing and problem solving skills. The path has also involved several aspects: psychological, perceptual, language and practical. Finally, we have always tried to activate the argumentation and conjecture processes to facilitate the transition from intuitive notions and operating levels to forms of deductive thinking and to abstract or virtual levels.

## **4 Conclusions and Future Work**

From the feedback of customer care questionnaires performed by the students and teachers involved, it is clear the high level of satisfaction, pushing us to continue these activities. The education project of MHS collected approval of principal Italian Mathematics associations and caused interest in institutional authorities at both local and national levels. The school environment welcomed it with enthusiasm so that the next year all over Italy some other high schools are starting. The next year activities will be enriched with an e-learning platform in order to support flipped teaching. Furthermore, already two high schools will install a laboratory of mathematics machines. On the one hand, the involvement of these machines will be intensified as an instrument of semiotic mediation inside the learning process. On the other hand, there will be an attempt to improve the linguistic skills. We will try to create a permanent pair between internalization of thoughts through the improvement of a practical intelligence and the exteriorization process of thoughts through the improvement of the

language, intended as a multimodal (including oral text, symbolic expressions and figurative representations) and multivariate (including a wide range of registers) system.

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