

# THE YOKE

## (Ethno)materials for math classes\*

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### ABSTRACT

We present some contributions to the identification of the existing processes in a mathematical artefact of the agricultural region of Trás-os-Montes and Alto Douro (in the northeast of Portugal) – the yoke.

The main aims of this study were to collect material on traditional jobs endangered in Portugal and analyse it, looking for mathematical contents to adapt its use in the classes of mathematics.

We report the implementation of tasks inspired by the yoke in mathematics' lessons of three classes of the 9th grade (students with 14-15 years old) in a school of Vila Real in the school year of 2007/2008.

This is a work in the field of ethnomathematics, focusing on the study of the mathematical knowledge and the know-how of a traditional job.

## 1 The yoke

A yoke is a wooden beam which is used between a pair of oxen to allow them to pull a load. There are several types used in different cultures and for different types of oxen. In Portugal there are two types of yokes (see figure 1): Yoke board (*Jugo de tábua*) and Yoke beam (*Jugo de trave*).



Figure 1: Yoke board<sup>1</sup> (on the left) and Yoke beam (on the right)

The yokes beam are all over the country, while the yokes board are characteristic of the western region (districts of Aveiro, Porto, Braga and Viana do Castelo). The yokes have peculiarities depending on the region.

*"Besides the ornamented yokes, there are also simple and modest yokes, who generally make the transition between those zones and higher elevations, except as*

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<sup>1</sup> Costa's drawing. All the other photos included in the paper were taken by the authors.

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*part of the Alto Douro and Trás-os-Montes, where we find the “molhelhas (...) leather and padded to give the horse a look full of surprises.” (Mattos, 1942, p. 68)<sup>2</sup>*

In Trás-os-Montes e Alto Douro there are three traditional types of yokes, namely the yoke of Vilar de Ossos – Vinhais, the yoke of Agarez – Vila Real and the yoke of S. Salvador de Viveiro – Boticas, illustrated in figure 2.



Figure 2: The three traditional yokes of Trás-os-Montes e Alto Douro

In this region of Portugal the yokes are used with “molhelhas” (see figure 3), a kind of pillows put between the oxen and the yoke.



Figure 3: A yoke with “molhelhas”

The motorization of agriculture made the use of animal's work a method obsolete. This is a strong reason for the disappearance of many traditional occupations related to agriculture. Nevertheless in some small farms of the region the use of tractors and other equipments cannot be used because of the characteristics of the land and not worth the investment (Hopfen, 1981).

This study about yoke makers continue other studies made by some of the authors about other traditional jobs of this region, such as wine coopers (Costa et al, 2008a), tin men (Costa et al, 2008b).

## 2 The yoke makers

The yoke makers were farmers or carpenters. Nowadays they only make repairs in yokes. This traditional job is endangered. We interviewed three yoke makers of the region of Trás-os-Montes e Alto Douro, namely from Justes, Samardã and Frieira.

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<sup>2</sup> Authors' traduction.

## 2.1. Methodology and collecting of data

In this study we used a qualitative methodology (Bogdan and Biklen, 1994) consisting of data collection done by the authors at the places where yoke makers live or work. This data collection was based on unstructured interviews recorded in audiovisual media, field notes and direct observation. The craftsmen interviewed have been possible within the local contacts provided by acquaintances. The data we present are transcripts of interviews, field notes, photographs as possible while respecting the manner in which they were recorded. For the preparation of the interviews we made a documentary analysis of the literature on this subject available in ethnographic studies (Mattos, 1942), (Oliveira, 1985), (Vasconcelos, 1938).

## 2.2. The interviews

In this section we present some parts of the interviews made to the three yoke makers.

**Marcelino Pereira** (see figure 4), 70 years old, was born at Justes. He had several jobs, but he always worked in agriculture. He is yoke maker for more than half a century. He summarized his work as yoke maker by saying: “I was born in it and I have a passion for it!”



Figure 4: Mr. Marcelino Pereira, a yoke maker

**José Domingos Costa** (see figure 5), 52 years old, was born at Samardã. He worked as yoke maker until get married. Since then he is carpenter and he still makes repairs in yokes. He had times that he went to mountain for two weeks to make yokes. As for his initiation as yoke maker, he said that he learned because of necessity: “*I was working on the field and the yoke broke ... I thought that I had to pay to others to repair it. So, I did it by myself. It was not well as it should be, but then I learned.*”



Figure 5: Mr. José Domingos Costa, a yoke maker

**Daniel Augusto Morais** (see figure 6), 75 years old, was born at Frieira. Mr. Daniel is retired from a life dedicated to carpentry. He still makes handicrafts.

*“I formerly did everything... Nowadays, I don’t work to anybody. I am doing a piece of the plow, but this is just for decoration.”*

He learned the art of carpentry with his father. In his words: *“My father was a carpenter and my brother too, but then my brother went to Brazil.”*

While still working, Mr. Daniel had to travel frequently to other villages to build farm tools. He used a motorcycle to carryout his tools. *“I bike in the rain and snow everywhere... I took this box full of tools and with this middle mold of a yoke.”*

The three yoke maker referred that the know-how based on comparison with a model (another yoke) was the “technique” used by them.



Figure 6: Mr. Daniel Augusto Morais, a yokemaker

### **3 The teaching experience**

In this section, we report the implementation of tasks inspired by the yoke in mathematics’ lessons of three classes of the 9th grade (students with 14-15 years old) in a school of Vila Real in the school year 2007/2008. This teaching experience was integrated in the Project *Ciência Viva VI* – number 771 (Costa et al, 2008).

#### **3.1. Aims, methodology and collecting of data**

The aims of the teaching experience were the:

- Identification of mathematical procedures used in traditional jobs of the region;
- Explanation, interpretation and register of the same framed in the mathematical abilities in the different school years;
- Creation, by the students, of educational resources (didactical material) that illustrate this knowledge and enable its divulgation and reproduction;
- Application of experimental methodologies.

In what concerns the methodology and the collecting of data we used a qualitative methodology (Bogdan and Biklen, 1994). The data was collecting by direct observation, the teacher diary and by the students’ productions.

#### **3.2. The teaching experience phases**

The teaching experience had two different phases: one of teaching planning and another of classroom implementation. The teacher also had to study similar educational experiences to plan his own, analyzing for instance (Gerdes, 2007), (Gerdes, 2008), (Moreira, 2004).

**Phase 1 – Lessons planning**

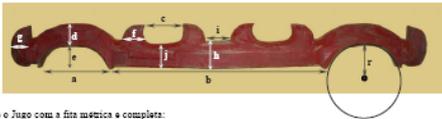
First the teacher analyzed the yoke makers' interviews and the mathematical potentialities of the yoke. After this, the teacher prepares tasks involving experimental methods and the yoke. The teacher constructs three worksheets (see figures 7, 8 and 9) for each yoke/class. Finally the teacher collects the three typical yokes of the region of the school (Trás-os-Montes e Alto Douro) and takes the yokes to the classes.



Ref. do Projecto n.º 771  
 "E se a Matemática transformasse a minha terra na capital do universo?"  
 (Singela homenagem ao algebrista José Morgado Júnior, natural de Pegarinho)

**O Jugo e as Circunferências**  
 Algumas coincidências

Jugo de Vilar de Ossos – Vinhais



Mede o Jugo com a fita métrica e completa:

a = \_\_\_\_\_ f = \_\_\_\_\_  
 b = \_\_\_\_\_ g = \_\_\_\_\_  
 c = \_\_\_\_\_ h = \_\_\_\_\_  
 d = \_\_\_\_\_ i = \_\_\_\_\_  
 e = \_\_\_\_\_ j = \_\_\_\_\_  
 r = \_\_\_\_\_

Descreve como procedeste para determinar r.

\_\_\_\_\_

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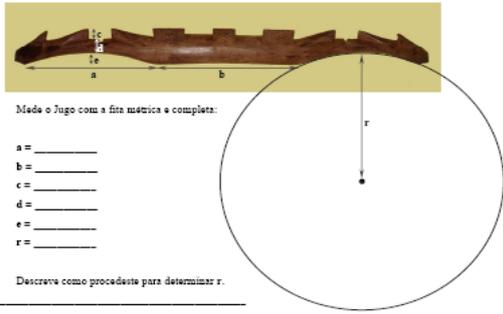
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 Algumas coincidências

Jugo de Agarez – Vila Real



Mede o Jugo com a fita métrica e completa:

a = \_\_\_\_\_  
 b = \_\_\_\_\_  
 c = \_\_\_\_\_  
 d = \_\_\_\_\_  
 e = \_\_\_\_\_  
 f = \_\_\_\_\_

Descreve como procedeste para determinar r.

\_\_\_\_\_

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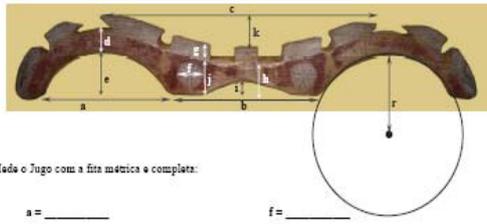
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Figure 7: Worksheet of Vilar de Ossos' yoke    Figure 8: Worksheet of Agarez' yoke

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*O Jugo e as Circunferências*  
*Algumas coincidências*

Jugo de S. Salvador de Viveiro - Boticas



Mede o Jugo com a fita métrica e completa:

- |           |           |
|-----------|-----------|
| a = _____ | f = _____ |
| b = _____ | g = _____ |
| c = _____ | h = _____ |
| d = _____ | i = _____ |
| e = _____ | j = _____ |
| k = _____ | r = _____ |

Descreve como procedeste para determinar r.

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9.º Ano, Turma: \_\_\_\_\_ Alunos a.: \_\_\_\_\_ ; \_\_\_\_\_ ; \_\_\_\_\_ ; \_\_\_\_\_

Figure 9: Worksheet of S. Salvador de Viveiro' yoke

**Phase 2 – Lessons implementation**

In the classroom, in groups, students and teacher explore the mathematics involved in the yoke (see figure 10) and construct final products that will be presented in a public meeting in the University of Trás-os-Montes e Alto Douro.



Figure 10: Students' study of the yoke

The research on teaching practices in each of the three yokes originated surprising results that students have summarized in their notes and final products, which images are reproduced in figures (see figures 11, 12, 13 and 14). These images illustrate the relations found between the three yokes and circumferences. OuvirLer foneticamente

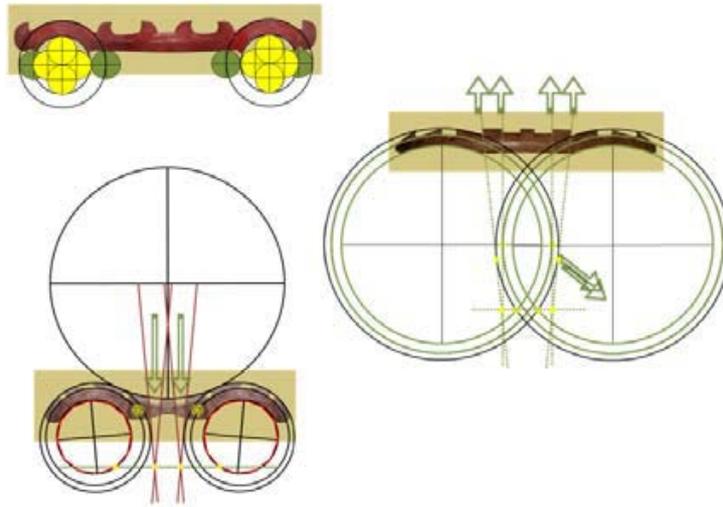


Figure 11: Students' final productions

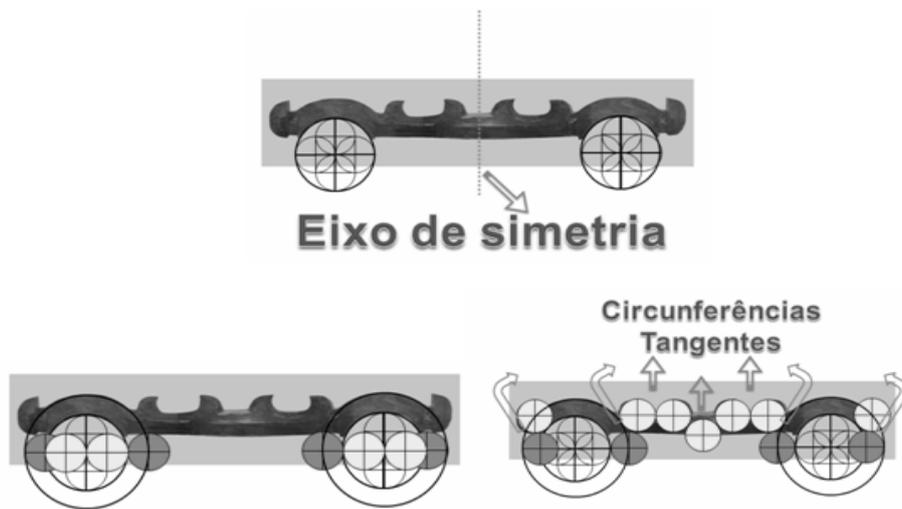


Figure 12: Students' final productions

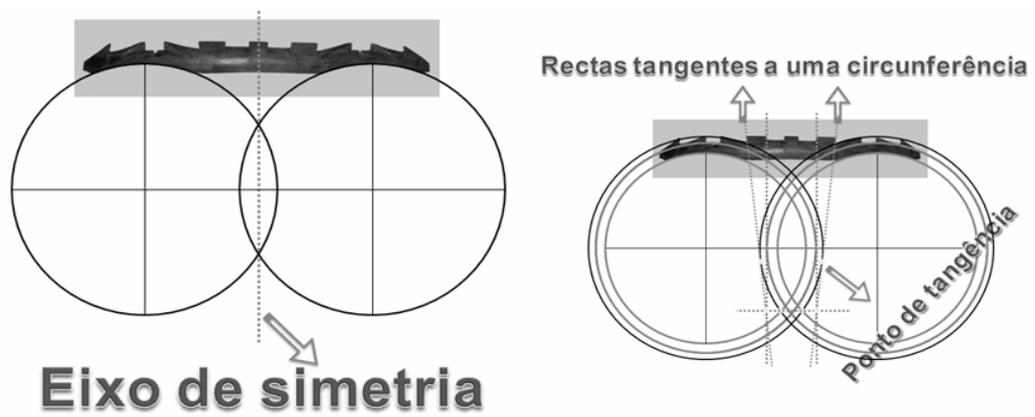


Figure 13: Students' final productions

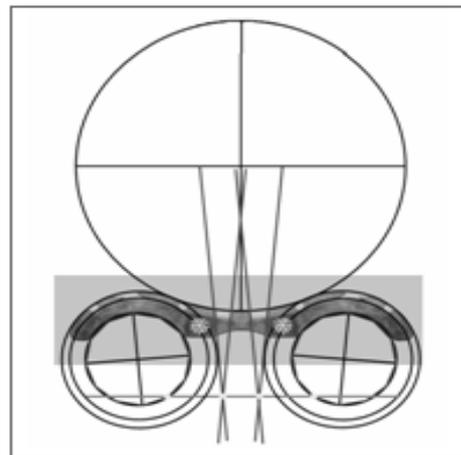
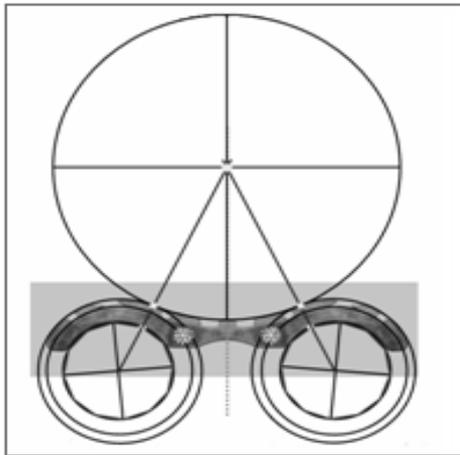
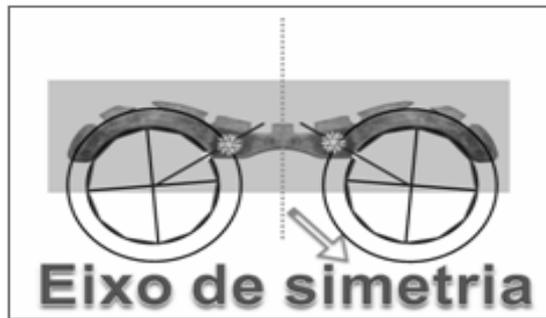


Figure 14: Students' final productions

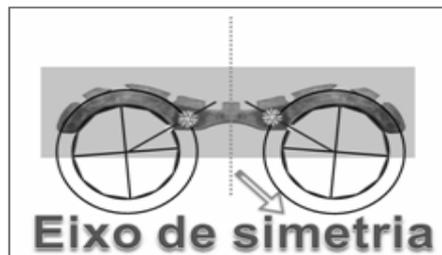
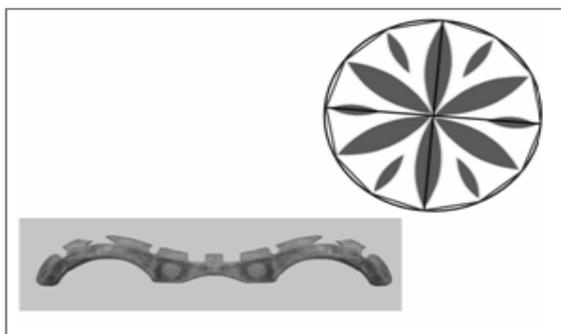


Figure 15: Students' final productions

This teaching experience allowed the students to study many contents of geometry, namely: angles, regular polygons, trigonometric ratios, circular arcs, circumferences, tangents' lines of circumferences, symmetry, displacements and rotations.

## 4 Final remarks

The yoke makers' interviews were gratifying for us, because of the way they welcomed us and made available to talk about their art.

The yoke makers proved to be thankful for the recognition, appreciation and divulgation that we would give to their work.

The study, of the processes used by yoke makers, allows us to identify the existence of mathematical processes which use measurements, symmetry and proportions. This fact is not new, because other studies conclude the same about other artisans (Palhares, 2008).

The use of experimental activities in the classroom motivated the students. They participated more actively than was usual.

The research done proved to be mathematically rich, creative, engaging and stimulating for students.

We consider that it is important that teachers should promote activities that could contribute to motivate students and the involving community, and that reinforce the cultural and scientific identity of the students.

With this teaching experience we verified that it was very important for students doing experimental activities. They improved their abilities of measurement and be aware of the importance of the rigor and accuracy of measurements.

The relations between the yoke and circles were not found in the processes used by yoke makers.

The collection of aspects related with traditional jobs contributes for a future memory of the region history.

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