PRACTICAL GEOMETRIES IN ISLAMIC COUNTRIES

The Example of the Division of Plane Figures

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ABSTRACT

The division of plane figures is a geometrical chapter developed in numerous works written in Arabic. In the extension of Greek practices, this chapter is also found in original developments in Islamic countries. The aim of the presentation is to show the diversity from several books of the Muslim Orient and Occident from the 9th century until the 14th century.
This diversity is first based on the multiple origins of the problems. They are linked, among others, to the practices of craftsmen, architects, or jurists. For example, jurists had to decide on the sale or the sharing of fields. To divide a geometrical figure in a certain number of similar figures is an important problem for the decorators who embellished palaces, *madrasas*, and other mosques and mausoleums. Moreover, these problems are illustrated in some writings of eminent geometers.
This diversity also expresses itself by the wealth of procedures of construction and resolution for which the whole mathematical knowledge is included.

1 Introduction

It is unfortunate that the history of scientific activities in Islamic countries is so poorly known to Europeans and this history repeats itself. Today, some European scholars (certainly ideologically tainted) neglect the original developments of science in the Islamic era and even deny their appropriation by Christian Europe from the 12th century.

Even though studies on the advent and development of scientific activities in Islamic countries are still incomplete or deficient, our goal in this paper is to give some general and major features of their history. Then, we discuss the interactions that have been established between scientific practices and several social, cultural, political, and religious aspects.

2 Sciences in Islamic Countries.

2.1 Generalities on the Scientific Development in Islamic Countries.

From Hegira (632), the Islamic countries, that is to say all regions dominated and unified by a single religion – Islam – gradually correspond to a huge Empire. It extends from the Pyrenees to Timbuktu (from North to South), and from Samarkand to Saragoza (from East to West). In this controlled and pacified geographical area, Islamic law is the canon law for the society. All roads, in particular those of trade, are free. In the history of scientific practices in Islamic countries, three major periods can roughly be distinguished[1].

The first one is a period of appropriation of the knowledge of the Ancients (Syrian, Persian, Sanskrit or Greek, of course). Scientific activities benefit from the fluidity of movement of men and books. The local scientific practices (weights and measure, calculation of inheritance, decorative art, astrology, for example) are not neglected and the

[1]For further details, see Djebbar, A., 2001, *L'âge d'or des sciences arabes*, Paris: Seuil.

scholarly knowledge consolidates them and introduces rational approaches. Arabic, the language of the *Qur'ān*, is needed as the language of scientific communication[2]. An important movement of translation grows, from the 8th century until the mid-10th century, to become acquainted with the whole knowledge of the Ancients[3]. From the first conquests of new territories, Islam held a leading role for the sciences.

The second period of the history of scientific practices in Islamic countries runs from the 9th century to the 12th century. It is the so-called "Golden Age of sciences in Islamic countries", corresponding to a period of scientific creation and development. When the knowledge from the Ancients was assimilated, scientists from Islamic countries taught, commented, and surpassed them. Many scientific foyers emerged, both in the East and in the West, such as in Baghdad, Samarqand, Cairo, Cordoba, but also in Kairouan, Nishapur, and Marrakesh. In addition to improving results inherited from the Ancients, innovations are observed, for example, in medicine, astronomy, and mathematics. New disciplines emerged such as trigonometry, algebra, and combinatorics in mathematics. These developments were widely promoted by several factors including the patronage of Princes and foremost that of the Caliph himself, by various social demands or by the non-intervention of religion in scientific practices.

The third period is not a period of decline as it has often been characterized. Original scientific researches continued to occur (in mathematics and astronomy) but they were more isolated in several parts of the Islamic area, both in the West with the Maghreb and Andalus and in the East with actual Iran. This period is characterized, from the late 12th century, by the disappearance of Arabic as the only language of scientific discourse. Indeed, three other languages competed with Arabic: Persian in the East and Hebrew and Latin in the West.

The men of science in Islamic countries, whatever their profile, took advantage of collective management and its contingencies to guide and deepen part of their production. We now discuss a few aspects of the "Islamic culture" (taken in its widest meaning) that are considered as an *impetus* of scientific research in Islamic countries.

2.2 Mathematics and Cultural Aspects in Islamic Era.

To be encouraged and developed, any scientific knowledge needs institutional support and cultural values. Science in Islamic countries is no exception. First of all, institutionally, we mention the strong political will of many successive Caliphs to support scientific research and teaching. Then, culturally speaking, several direct or indirect evidences guarantee the existence of interaction between the scholarly science and the know-how of artists and crafstmen. We shall even see that some of these evidences give the proof of a certain stimulation of scholars by or for craftsmen.

The Umayyadds (661-750) and the first Abbasids (especially between 750 and 850) expressed the same desire by supporting scientific activities. The Caliph al-Ma'mūn (813-833) is probably the more significant. To provide the library *Bayt al-ḥikma* [House of Wisdom] in Baghdad, he interceded with Leo V (813-820), Emperor of Byzantium, to obtain books on philosophy and science. He also supported delegations of scholars in Asia

[2]Thus, due to an abuse of language, science in Islamic countries is sometimes called « Arabic science ». The term «arabic» must have been understood in the meaning of the language used to write and teach the science. It does not refer to geographic, cultural or religious origins.

[3]Gutas, D., 1998, *Greek Thought, Arabic Culture*, New York: Routledge.

Minor and Cyprus to bring books written in Greek. He organised the measurement of the diameter of Earth. He gave assignments to scientists in order to determine the geographical locations of various events described in the *Qur'ān*. This same Caliph encouraged al-Khwārizmī to "*compose a short book on algebra and muqabala*[4]*"*, namely *Mukhtaṣar fī ḥisāb al-jabr wa l-muqābala* [Compendium on calculating by completion and reduction], which can be regarded as the official birth of Algebra as a new field of knowledge. This kind of patronage was still present at least until the first half of the 15th century. This is confirmed by the astronomer and mathematician al-Kāshī (d. 1429) in his correspondence with his father. Member of the scientific staff of Ulūgh Beg (1394-1449) in Samarqand, one of the most important scientific foyer of the Muslim Orient, he wrote on the construction of a *mihrāb*[5] according to the wishes of the Sultan: "*His Majesty [once] said: ''We would like to make a hole in the wall of a* mihrāb *in such a way that the sun may shine through that hole for a short while at the afternoon [prayer] time both in summer and in winter. That single hole must be round from inside, but from the outside it must be in such a way that sunshine cannot pass through it at times other than the afternoon [prayer time]. This [royal wish] had been [already] expressed before my arrival, and nobody had been able to realize it; [but] when I came [here], I did this also*[6]."

This last evidence allows us to evoke the idea that some of the scientific production (research and teaching) can be considered as replies to individual or collective societal needs. These responses are directed to several practitioners such as architects, craftsmen decorators and, according to al-Khwārizmī himself, "*inheritance, legacies, partition, law-suits, and trade, and in all their dealings with one another, or where the measuring of lands, the digging of canals (...) are concerned*[7]". The duality between the scholar and the practitioner is not widely known but is nevertheless real. It is illustrated by three distinguished scholars of Islamic countries who provide evidence on meetings between mathematicians and craftsmen, which can be considered as a forum to discuss methods for designing ornamental patterns in several materials (wood and tile, for example). The first one, Abū l-Wa'fā' al-Buzajanī (940-998), is a Persian astronomer and mathematician who worked in Baghdad from 959. In his book *Kitāb fīmā yaḥtāju ilayhi as-sanī*^{c} *min al-a*^{c}*māl al-handasiyya* [Book on What is Necessary from Geometric Constructions for the Craftsmen], he specified that he participated in such a meeting in which was discussed the construction of a square from three equal squares[8]. The second one is the famous Persian mathematician and poet: ^{c}Umar al-Khayyām (1048-1131). In an untitled work, he reported a solution of a problem (by using a cubic) settled in a meeting, which could be in Isfahan with mathematicians, surveyors and craftsmen[9]. Al-Khayyām's collaboration with craftsmen does not seem to be limited to this. Indeed, he could be the designer of the

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[4]Rashed, R., 2007, *al-Khwārizmī, le commencement de l'algèbre*, Paris: Blanchard, p. 95.
[5]A *mihrāb* is an alcove in the mosque indicating the direction of prayer.
[6]Bagheri, M.. 1997, "A newly found letter of al-Kāshī on Scientific Life in Samarkand", *Historia Mathematica* **24**, 241-256.
[7]R. Rashed. *al-Khwārizmī...*, p. 95.
[8]Abū l-wafā' al-Būzajānī, 1979, *Kitāb fīmā yaḥtāju ilayhi aṣ-ṣānī*^{c} *min a*^{c}*māl al-handasiyya* [Book on What is Necessary from Geometric Constructions for the Craftsmen], Introduction and critical edition by Al-^{c}Ali, S.A., Baghdad: Imprimerie de Bagdad, p. 145.
[9]Amir-Moez, A., 1963, "A Paper of Omar Khayyam", *Scripta mathematica* **26**, 323-337.

North Dome Chamber of the Friday Mosque of Isfahan[10]. The third and last evidence dates from the 15th century. In a letter to his father, al-Kāshī described its resolution of a problem settled during a meeting between craftsmen, mathematicians and other dignitaries[11]. He also showed an excellent knowledge of architects, designers, and other craftsmen by measuring domes and *muqarnas* in his major *Miftāḥ al-ḥisāb* [Key of Arithmetic][12].

Now, we will illustrate the close relationship between the speculative developments of scholars and real issues of practitioners, whatever their original corporation, with a kind of geometrical problems: the division of plane figures.

3 The problems of the Division of Plane Figures

Our purpose is not to give a historical background of this kind of problem[13]. Nevertheless, it is necessary to detail what we understand by "division of plane figures". It is an old and very diverse mathematical chapter with problems issued by Old Babylonian scribes dating from 1800BCE. Typically, cutting off or dividing a plane figure is sharing it according to several constraints set *a priori*. These constraints are related to geometric properties of the transversal(s) or desired figures. And, they are on magnitudes with several conditions on ratios given on parts obtained after the division. For example, we have to divide a rhombus in two parts according to a given ratio by a line parallel to one of its sides. The problems can also consist in dividing a (or several) given figure(s) in order to acquire an (or many) other(s) respecting conditions of similarity for example. In our present paper, the problems of inscription (or circumscription) and the cutting off of figures necessary to the measurement (as the triangulation, for example) are not taken into account even if they are quite important in their cultural aspects (ornamentation and surveying, for example).

It is not difficult to guess that these kinds of problems could easily be related to professional activities in everyday life. Craft or legal traditions of such sophistication involved a significant amount of technical and mathematical knowledge. Even if this knowledge was above all transmitted from master to apprentice or from a brother to another in a same corporation rather than being written down, it is possible to illustrate it from books dealing with mathematics. First of all, we present briefly the scholarly tradition. Then we will devote our purpose to two main cultural topics: inheritance and ornamentation.

3.1 Division of Plane Figures and Speculative Geometry[14]

The scholarly orientation of the division of plane figures in Islamic science is roughly characterized by the reception and the appropriation of an Euclid's text. First of all,

[10] Özdural, A., 1998, "A Mathematical Sonata for Architecture: Omar Khayyam and the Friday Mosque of Isfahan", *Technologie and Culture* **39**, 699-715.

[11] Kennedy, E.S., 1960, "A Letter of Jamshīd al-Kāshī to His Father", *Orientalia* **29**, 191-213. p. 198

[12] Dold-Samplonius, Y., 1992, "Practical Arabic Mathematics: Measuring the Muqarnas by al-Kāshī", *Centaurus* **35**, 193-242.

[13] Moyon, M., 2008, *La géométrie pratique en Europe en relation avec la tradition arabe, l'exemple du mesurage et du découpage : Contribution à l'étude des mathématiques médiévales*, PhD in Epistemology and History of Sciences supervised by Djebbar, A., University of Lille1.

[14] We only give major features. We have largely studied this subject in our thesis. Moyon, M., 2008, *La géométrie pratique…*

as-Sijzī, one of the most prolific Islamic geometers in the tenth century, wrote an opuscule which explicitly refers to the *Kitāb Uqlīdis fī l-qismat* [Book of Euclid on the divisions][15]. It could be a partial Arabic translation of *On the divisions*, a lost book of Euclid. As-Sijzī introduced thirty-five problems, of which only four were proved. The others were considered as easy by the Persian mathematician[16].

Another author, Muḥammad al-Baghdādī, proposed the same kind of problem in a scholarly book, the *De superficierum divisionibus Liber*, inspired by the previous one. Only the Latin translation, completed in the twelfth century, has survived. The author, probably active between the tenth and the twelfth century in the Muslim Orient, remains unknown.

In these two books, the statements are very general and the given proofs are constructed according to the Euclidean model (hypothetico-deductive). They are based on the *Elements*, in particular on Books I, II, V and VI.

3.2 The Division of Plane Figures as a Source of Decorative Patterns.

In this section, we present a unique example from the *Kitāb fīmā yaḥtāju ilayhi as-sanī^c min al-a^{c}māl al-handasiyya* of Abū l-Wafā‘ that we have already mentioned. Indeed, in the tenth chapter, he gives us valuable evidence comparing, for a same problem, the empirical approach of the craftsman to the speculative one of the geometer[17]. This problem concerns the division of three equal squares in order to compose an other one.

In this context, Abū l-Wafā‘ opposes two methods of the decorator craftsmen based on know-how, intuition, experimentation, and trial and error to the application of a theoretical result established by mathematicians disconnected from design and material reality. *“A group of geometers and craftsmen were wrong in the matter of these squares and their assembling, the geometers because they have little experience in practice, and the craftsmen because they lack knowledge of proofs. The reason is that, since the geometers do not have experience in practice, it is difficult for them to approximate according to the requests of the craftsman what is known to be correct by proofs by means of lines. And, the purpose of the craftsman is what makes the construction easier for him, and correctness is shown by what he perceives through senses and by observation and he does not care about the proofs of the imagined thing and <the correctness of> the lines. (...)The geometer knows the correctness of what he wants by mean of proofs when he is the one who has extracted the notions on which the craftsman and the surveyor work. But, it is difficult for him to apply the proofs to a construction when he has no experience with the work of the craftsman and the surveyor. If the most experienced among these geometers are asked about something in dividing figures or something in multiplying lines, they hesitate and they need a long time to think. Perhaps it is easy for them but perhaps it is difficult for them and they do not succeed in its construction*[18].”

[15]Hogendijk, J.P., 1993, “The Arabic version of Euclid’s On Divisions”, in *Vestigia Mathematica, Studies in Medieval and Early Modern Mathematics in Honor of H.L.L. Busard,* M. Folkerts & J.P. Hogendijk (eds), Amsterdam-Atlanta: Rodopi, pp. 143-157; p. 149

[16]Hogendijk, J.P., 1993, “The Arabic…”, p. 159.

[17]Abū l-Wafā’ did not forget problems settled and solved in a more scholarly way in the chapter eight for triangles and the ninth one for quadrilaterals. Abū l-Wafā’. *Kitāb fīmā…*, p. 103-127.

[18]Abū l-Wafā’. *Kitāb fīmā…*, p. 144-145.

Abū l-Wafā‘ also acknowledged that some of the methods used by craftsmen were wrong even though they seemed correct in appearance to an observer uneducated in scholarly geometry. He then explained some of the methods known to craftsmen in order to distinguish those that are correct (that is to say demonstrable by geometers) from others. The Persian mathematician also proved why certain methods are inaccurate[19]. Once he studied the methods of craftsmen, he gave his own construction that inspires a new decorative pattern[20].

3.3 Division of Plane Figures and Islam: the Inheritance

Rituals and religious prescriptions in the respect of *al-qur'ān* and *al-ḥadīth* [words of the Prophet Muḥammad] involve technical problems that encourage scientific researches. We cite, for example, the determination of *qiblā* (the direction of Mecca), the lunar month, and especially the observance of *Ramadan* for thirty days a year, or the calculation of the exact time of the day for all the prayers.

Here, we focus on another aspect of the Islamic law : the *ᶜilm al-farā'id* [science of partitions of the inheritance]. In addition to the questions on arithmetic or algebra[21], this science also raises geometrical problems including the sharing of land between partners or beneficiaries. All of these problems seem to be issues (at least in form) presented to the surveyor when fields are separated. They can be found in books of geometry or in textbooks on reckoning in the chapter dealing with *misāḥa* [measurement]. We present here three examples of sharing of lands between co-owners or eligible parties by creating a way-in for each of them. These three examples have different solutions[22].

The first one is a problem from the last part "On the construction of a way"[23] of the ninth chapter dealing with the divisions of quadrilaterals of the geometric book written by Abū l-Wafā‘ already encountered. The problem is solved with a geometrical construction based on a typical Euclidean style of the *Elements*. No proof is given to justify the exactness of the construction[24].

The second example is extracted from the book of Ibn Ṭāhir al-Baghdādī (d. 1037), an 11th-century mathematician, intitled *Takmila fī l-ḥisāb* [The Completion of Arithmetics][25]. The resolution given by the mathematician of Baghdad is algorithmic. Indeed, he established a general algorithm in order to use it in a special case where it is necessary to share a rectangular field between three brothers with an access for the different parts.

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[19]See appendix 1.

[20]Özdural, A., 2000, "Mathematics and Arts: Connections between Theory and Practice in the Medieval Islamic World", *Historia Mathematica* **27**, 171-201.

[21]Al-Khwārizmī devotes a substantial part of his *Mukhtaṣar* to the resolution of problems on legacies. Rashed, R.. *Al-Khwārizmī…*, p. 232-291. For further details on this topic, see Laabid, E., 2006, *Les techniques mathématiques dans la résolution des problèmes des partages successoraux au Maghreb médiéval : l'exemple du Mukhtaṣar d'al-Ḥūfī (m.588/1192)*, PhD in History of Mathematics supervised by Lamrabet, D. and Djebbar, A., University of Rabat.

[22]See appendix 2.

[23]Abū l-Wafā'. *Kitāb fīmā…*, p. 127-132.

[24]In this section, Abū l-Wafā' exposes five problems (two in a square, two in a triangle, one in a trapezium). Note that the last three constructions are erroneous.

[25]Ibn Ṭāhir al-Baghdādī, 1985, *At-takmila fī l-ḥisāb* [The completion of Arithmetics], Introduction and critical edition by Saïdan, A.S., Koweit: Ma'had al-makhṭūṭāt al-'arabīyya.

The third and last example can be read in the *Kāfi fī l-ḥisāb* [The Sufficient in Arithmetic] of al-Karajī (d. 1023)[26]. This example is interesting for two main reasons. First of all, the statement of the situation seems dictated by Islamic law. Consequently, this leads to a strange mathematical problem with a sharing between the half, the third and the fourth! This is not an isolated case in the calculations of inheritance[27]. The mathematical solution takes into account this situation and the beneficiaries must accept a smaller part than that given by Islamic law. Secondly, it is the occasion for the Persian mathematician to use the objects and the operation characteristic of Arabic algebra.

4 Conclusion

Division of plane figures is indeed an interesting geometric practice to illustrate the scientific development in Islamic countries. At first, it seems to be based on the local tradition with traditional skills. Actually, however, it is the appropriation of the knowledge of the Ancients, the quest for rationality, and the use of new disciplines that progressively provide answers to these problems in everyday life.

Even if scientists and craftsmen of Latin Europe have different cultural and religious needs as their counterparts in Islamic countries, the problems of division of plane figures are part of the many practices and knowledge that Latin scholars approppriated from the 12th century. We have mentioned the Arabic-Latin translation of the text of Muḥammad al-Baghdādī. We also discussed the *Liber Embadorum* of Platon de Tivoli, the Latin version of the *Hibbur ha-Mesihah we-ha-Tisboret* of the Hebrew scientist Abraham Bar Ḥiyya. These books both dedicate a whole chapter to this topic, the study of which highlights a dual origin: scholarly, but also practical with sharing between beneficiaries. Finally, it is also the case of the fourth distinction of the famous *Practica geometriae* written by Fibonacci in the 13th century that will influence subsequent treaties of geometry.

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[26]Al-Karajī, 1986, *Al-Kafī fī l-ḥisāb* [The Sufficient in Arithmetic], Introduction and critical edition by Chalhoub, S., Alep: Institut d'Histoire des Sciences Arabes.
[27]Laabid, E., 2006, *Les techniques mathématiques* ..., p.41.

APPENDIX 1

Abū'l Wafā': *On the composition and division of squares*[28].

One of the craftsmen placed one of the squares in the center. And he bisected the second by means of the diagonal and he placed the halves on two sides of the <first> square. And he drew two straight lines from the center of the third <square> to two of its angles, not on one diagonal and a line from the center to the midpoint of the opposite side of the triangle which came with the two <previous> lines. Thus the square is divided into two trapezia and a triangle.
Then he placed the triangles below the first square and he placed the two trapezia above it. He combined the two longer sides <of the trapezia> in the middle. Thus he obtained a square as in this figure (fig. 1)

(Abū l-Wafā' said): As for the figure which is constructed, it is in the imagination, and someone who has no experience in the art and in geometry see that it is correct. But if we show it to him, he knows that it is false.
The fact that he imagines that it is correct is explained by the correctness of the angles and the equality of the sides. The angles of the square are correct, each of them is right, and as for the sides, they are equal. Because of this, he imagines that it is correct.
The angles of the <three> triangles – B, G and D – which are the angles of the square, are all right. And the fourth angle is assembled from the two angles which are all <equal to> half a right <angle>, they are the angles of the <two> trapezia.
The sides are straight and equal because each of these sides is assembled from a side of one of the squares and half its diagonals, which are equal. The fact that they are straight with the assembling is clear because the angles gathered at the meeting points of the lines are all equal to two right <angles>.
The three angles which are at point G are equal to two right <angles> because they are one angle of a square and two angles of a triangle which are, each of them, equal to half a right angle. And it is the same for angle T.
Angle I, meanwhile, has two angles, one of them is the angle of the triangle, that is half a right <angle>, and the other is the angle of the trapezium, that is one right <angle> and half <a right angle>. And it is the same for the two angles which are at point K.
As the angles are right and the sides are straight <and equal>, for everybody, it appears that a square has been constructed from three squares. And they do not realize the place where the error is introduced.
This is clarified for us if we know that each side of this square is equal to the side of one of the <first> squares and half of its diagonal. It is not permissible that the side of the square composed of three squares has this magnitude. Indeed, it is greater than that. And that is because if we consider the side of each <first> square <equal to> ten cubits to make it easy for the student, the side of the square composed of three squares is approximately seventeen cubits and one-third <of one cubit>. And the side of this

[28]Abū l-Wafā'. *Kitāb fīmā…*, p.146-148.

square is seventeen cubits and half of one-seventh [of one cubit], and between them, there is a gap.
What's more, when we bisected <the first> square ABGD and we placed each half of it next to the side of the square A, the diagonal of the square BG falls on two lines, HY and TK. And, it is not permissible that it falls on them for two reasons. One of them is that the diagonal of square BG is not expressible whereas line HY is expressible and it is as the side of square BG and half of it. The second is that it is less than that because the diagonal of square BG is approximately <equal to> fourteen and one-seventh, and side HY is <equal to> fifteen. Thus, the incorrectness of this division and this assembling clearly appears.

APPENDIX 2.

Abū l-Wafā'[29]

And if someone says: how divide a square ABGD into three equal parts and establish between them a path, whose width MQ is known, between two of the equal parts.
We extend GA towards I.
We construct AI equal to GM.
We extend BA towards E.
We construct <the circle> with center M and radius ME, and the circle intersects line BE (sic BA) at point E.
We join GE.
We cut off line ER from line GE equal to line GM.
We draw from point R line RHL parallel to line BAE.
And from two points M <and> Q, <we draw> two lines MT and QK both parallel to AG.
Thus there are equal surfaces MGHT, KLDQ, ABLH. And this is the figure for it. (fig. 2)

Ibn Ṭāhir al-Baghdādī[30]

And if we want to fit out a path in a land with right angles, or equal sides, or unequal its length or its width and that the land be divided between three people, or four people, or five people or whatever [the number of people], the method for it is to multiply the side on which we want to fit out the width of the path by the number of shares by which the land is divided, for example shares of the sons, of the daughters, of the two parents and of the husband. We subtract from it the width of the path and the rest is the divisor. Then, we multiply the area of the path by the number of heirs, minus the part of one who had charge of the path.
The result of the division is the length of the path. And when we know the length and the width of the path, the rest of the land can be shared between them according to <the rules of> sharing of God the Almighty.
Example of this: a land twenty by thirty that we want to divide between three brothers establishing, between them, a path whose width is two cubits but establishing the path from the thirty. We need to know how must be its length. We multiply thirty by three, and it results ninety. We subtract from it the width of the path and it is two cubits. It remains eighty-eight. This is the divisor that we keep. Then, we multiply the area by two, and this

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29 Abū-l-Wafā', *Kitāb fīmā ...*, p. 128-129.
30 Ibn Ṭāhir al-Baghdādī. *At-takmila…*, p. 372-373.

is the number of son minus one, it will be one thousand and two hundred. It is necessary to multiply by two because the path is used for the passage of two people. We divide one thousand and two hundred by eighty-eight. The result of the division is the length of the path. And this is the figure for it (fig. 3).
And if the division is between two sons and one daughter, it will be between five parts. If it is between two daughters and one son, it will be between four parts. And everything that reaches to you in this chapter <resolves> it according to this method and it will be its solution. And this is the figure for it. The length of the figure is thirty, its width is twenty, the length of the path is thirteen and seven elevenths of cubits, the width of the path is two cubits, the area of the path is twenty-seven and three elevenths and the area of the whole land is six hundred. If the area of the path is taken off, it remains five hundred and seventy-two and eight eleventh of cubits.
The verification of its <validity consists in> measuring the part of the one who is below and in seeing if it is equal to the part of each of them. If the area of >the part of> the one is below is equal to the part of each of them, we will know that <the result> is correct. If it is different from them, it will be the opposite.

Al-Karajī[31]

If someone says: you have a rectangle whose length is twenty *bāb*[32] and width ten *bāb*. Divide it between three people : one half for one of them, one third for another and one fourth for another in order to there is, in his center, a way whose width is two *bāb* that entries of the three quotas lead by the length, one by the front, another by the right and another by the left so that the quota of the owner of the third is on the front according to this figure (fig. 4).
The computation for that is to call the length of the way thing. You multiply it by the width of the way, and it results two things and this is the measurement of the way. And you consider the remaining eighteen <to divide> into two parts between the owners of half, and of fourth because they take their part from the right of the way, and from his left. One of the two parts is twelve. This is the width of the part of the owner of half.
And the remaining six is the width of the part of the owner of fourth. And the length of each <part> is the length of the way, and this is the thing. It results the measurement of the part of the owner of half : twelve things. And the measurement of the part of the owner of fourth is six things. And from this computation, the measurement of the part of the owner of third has to be eight things, and the measurement of the way is two things, and the whole measurement of this surface is twenty-eight things. And this is equal to two hundred.
And the only thing is equal to seven *bāb* and one-seventh *bāb*. And this is the length of the way. And it remains the width of the part of the owner of third from the whole width of ten *bāb*: two *bāb* and six-seventh *bāb*.

[31]Al-Karajī, *Al-Kafī* …, p. 202-204.
[32]*bāb* is an unit of measurement used in Islamic countries in both West and East.

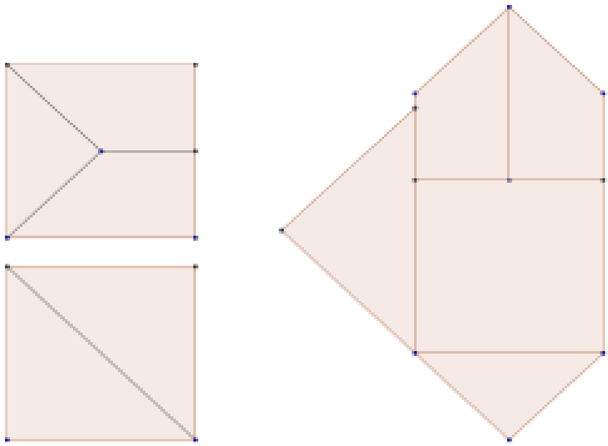

Figure 1 : Abū l-Wafā’

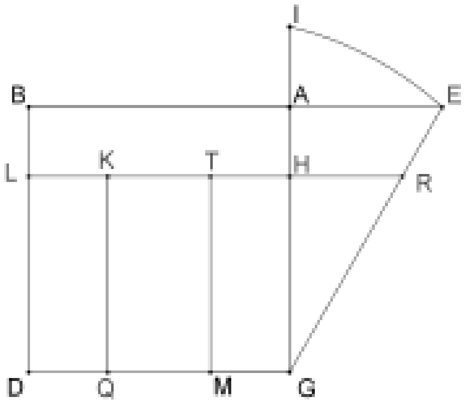

Figure 2 : Abū l-Wafā’

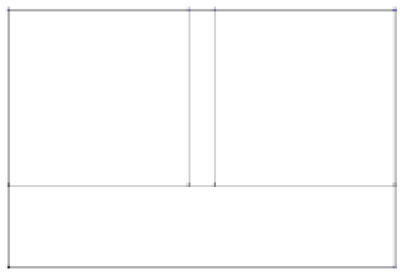

Figure 3 : Ibn Ṭāhir al-Baghdādī

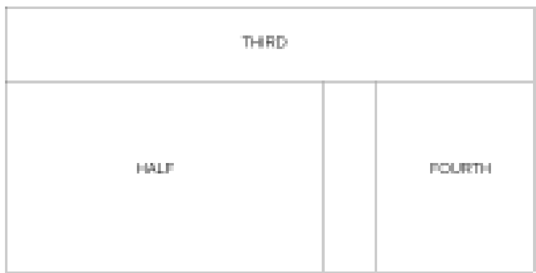

Figure 4 : al-Karajī